## REFERENCES

1. A. Bejan, *Convection Heat Transfer*. Wiley, New York (1984).
2. P. Cheng and W. J. Minkowycz, Free convection about a vertical flat plate embedded in a saturated porous medium with application to heat transfer from a dike, *J. Geophys. Res.* **82**, 2040–2044 (1977).
3. P. Cheng, Geothermal heat transfer. In *Handbook of Heat Transfer Applications* (Edited by W. M. Rohsenow, J. P. Hartnett and E. N. Ganic) (2nd Edn), Chap. 11. McGraw-Hill, New York (1985).
4. A. C. Mueller, Process heat exchangers. In *Handbook of Heat Transfer Applications* (Edited by W. M. Rohsenow, J. P. Hartnett and E. N. Ganic) (2nd Edn), Chap. 4, Part 2. McGraw-Hill, New York (1985).
5. A. Bejan, Theory of heat transfer from a surface covered with hair, Report DU-AB-8, Dept Mech. Engng and Mater. Sci., Duke University, Durham, North Carolina (February 1989).


*Int. J. Heat Mass Transfer.* Vol. 33, No. 1, pp. 209–215, 1990
Printed in Great Britain
0017-9310/90 $3.00+0.00


# Coupled heat and mass transfer from a sphere buried in an infinite porous medium

F. C. LAI and F. A. KULACKI†

Department of Mechanical Engineering, Colorado State University, Fort Collins, CO 80523, U.S.A.

(*Received* 30 *November* 1988 *and in final form* 9 *May* 1989)

## INTRODUCTION

WITH THE knowledge accumulated from the previous studies on convective heat transfer in porous media, considerable attention has now turned to a more sophisticated problem that takes into account the mass transfer effects. The phenomenon, which is sometimes referred to as 'double-diffusive' or 'thermohaline' convection in geophysical fluid mechanics, has many important applications in energy-related engineering problems, for example, the migration of moisture in fibrous insulation, the spreading of chemical pollutants through water-saturated soil, the cooling of nuclear reactors and the underground disposal of nuclear wastes.

Nield [1] made the first attempt to study the stability of flow in horizontal layers with imposed vertical temperature and concentration gradients for coupled heat and mass transfer by natural convection in a porous medium. Bejan and co-workers [2–5] conducted a series of investigators to study these effects on natural convection for various geometries. In a recent study, Poulikakos [6] extended the results by Bejan [7] to consider buoyancy induced heat and mass transfer from a concentrated source in an infinite porous medium.

The purpose of this study is to analyze another practically important problem of natural convection induced by the combined action of temperature and concentration gradients from a buried sphere. The approach is parallel to that of Poulikakos [6], however, more complicated boundary conditions, i.e. combination of different thermal and concentration boundary conditions, are considered. Emphases have been placed on a fundamental examination of these effects on the flow, temperature and concentration fields.

## FORMULATION

Consider a sphere of radius $a$ buried in an infinite porous medium. For heat and mass transfer driven by buoyancy effects, the governing equations based on Darcy's law are simplified by introducing the stream function such that they are given by

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\Psi}{\partial R^2}=Ra\left[\left(\cos\theta\frac{\partial\Theta}{\partial\theta}+R\sin\theta\frac{\partial\Theta}{\partial R}\right)-N\left(\cos\theta\frac{\partial C}{\partial\theta}+R\sin\theta\frac{\partial C}{\partial R}\right)\right] \quad (1)$$

$$\frac{1}{R^2\sin\theta}\left(\frac{\partial\Psi}{\partial\theta}\frac{\partial\Theta}{\partial R}-\frac{\partial\Psi}{\partial R}\frac{\partial\Theta}{\partial\theta}\right)=\left[\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Theta}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)\right] \quad (2)$$

$$\frac{1}{R^2\sin\theta}\left(\frac{\partial\Psi}{\partial\theta}\frac{\partial C}{\partial R}-\frac{\partial\Psi}{\partial R}\frac{\partial C}{\partial\theta}\right)=\frac{1}{Le}\left[\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial C}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial C}{\partial\theta}\right)\right] \quad (3)$$

where temperature and concentration have been non-dimensionalized as follows:

$$\Theta=\frac{T-T_\infty}{T_0-T_\infty},\quad \text{constant temperature}$$
$$\frac{T-T_\infty}{q/ka},\quad \text{constant heat flux}$$
$$C=\frac{c-c_\infty}{c_0-c_\infty},\quad \text{constant concentration}$$
$$\frac{c-c_\infty}{m/Da},\quad \text{constant mass flux.} \quad (4)$$

The subscripts 0 and $\infty$ denote the condition at the surface of the sphere and at infinity, respectively.

Four different cases are considered in the present study:

(1) a sphere of constant temperature and concentration;
(2) a sphere of constant heat flux and mass flux;
(3) a sphere of constant temperature and mass flux;
(4) a sphere of constant heat flux and concentration.

Therefore, the boundary conditions can be summarized as follows:

at $R = 1$,

$\Theta = 1$ for constant temperature case

$\frac{\partial\Theta}{\partial R} = -1$ for constant heat flux case

$C = 1$ for constant concentration case

$\frac{\partial C}{\partial R} = -1$ for constant mass flux case

$$\frac{1}{R^2\sin\theta}\frac{\partial\Psi}{\partial\theta}=0; \quad (5)$$

† Author to whom correspondence should be addressed.

as $R \to \infty$,

$$\Theta = 0 \quad C = 0$$

$$\frac{1}{R^2 \sin\theta}\frac{\partial \Psi}{\partial \theta} = 0 \quad \frac{1}{R\sin\theta}\frac{\partial \Psi}{\partial R} = 0; \tag{6}$$

at $\theta = 0, \pi$,

$$\frac{\partial \Theta}{\partial \theta} = 0 \quad \frac{\partial C}{\partial \theta} = 0$$

$$\frac{1}{R\sin\theta}\frac{\partial \Psi}{\partial R} = 0 \quad \frac{\partial}{\partial \theta}\left[\frac{1}{R^2\sin\theta}\frac{\partial \Psi}{\partial \theta}\right] = 0. \tag{7}$$

In the above equations, the Rayleigh number, $Ra$, is given by

$$Ra = \begin{cases} \dfrac{Kg\beta_T(T_0 - T_\infty)a}{\nu\alpha} & \text{for constant temperature} \\ \dfrac{Kg\beta_T q a^2}{\nu\alpha k} & \text{for constant heat flux} \end{cases} \tag{8}$$

and the Lewis number, $Le$, is

$$Le = \frac{\alpha}{D}. \tag{9}$$

Parameter $N$ measures the relative importance of mass and thermal diffusion in the buoyancy driven flow and is defined by

$$N = \frac{\beta_c(c_0 - c_\infty)}{\beta_T(T_0 - T_\infty)} \quad \text{for case 1}$$

$$N = \frac{\beta_c(m/D)}{\beta_T(q/k)} \quad \text{for case 2}$$

$$N = \frac{\beta_c(c_0 - c_\infty)}{\beta_T(q/ka)} \quad \text{for case 3}$$

$$N = \frac{\beta_c(m/Da)}{\beta_T(T_0 - T_\infty)} \quad \text{for case 4} \tag{10}$$

where $\beta_T$ and $\beta_c$ are the coefficients of thermal expansion and concentration expansion, respectively; $q$ and $m$ are heat and mass flux; $k$ and $D$ are thermal conductivity and mass diffusivity. It is clear that $N$ is zero for pure thermal driven flow, infinite for mass driven flow, negative for aiding flow and positive for opposing flow.

## ANALYSIS

In the limit of small Rayleigh number ($Ra \to 0$), an analytical solution to the problem stated in the preceding section is obtained by means of a standard perturbation analysis. The approach assumes power series expression in $Ra$ for $\Psi$, $\Theta$ and $C$

$$\Psi = \Psi_0 + Ra\,\Psi_1 + Ra^2\,\Psi_2 + \dots \tag{11}$$

$$\Theta = \Theta_0 + Ra\,\Theta_1 + Ra^2\,\Theta_2 + \dots \tag{12}$$

$$C = C_0 + Ra\,C_1 + Ra^2\,C_2 + \dots \tag{13}$$

where $\Psi_i$, $\Theta_i$ and $C_i$ (with $i = 1, 2, \dots$), which are functions of $R$ and $\theta$, are obtained by substituting expressions (11)–(13) back to equations (1)–(3) and solving the equations resulting from collecting terms containing the same power of $Ra$. The solution procedure is straightforward, therefore only the final results for stream function, temperature and concentration up to the second-order convection correction are presented.

The zeroth-order solutions, $\Psi_0$, $\Theta_0$ and $C_0$, correspond tc the state of pure diffusion and are given by

$$\Psi_0 = 0 \tag{14a}$$

$$\Theta_0 = \frac{1}{R} \tag{14b}$$

$$C_0 = \frac{1}{R}. \tag{14c}$$

Although they have the same form for all four cases, however, it should be noted that the temperature and concentration are nondimensionalized with different parameters and the definition of the Rayleigh number is also different for each case.

*Case* 1. *A sphere of constant temperature and concentration*

The first-order solutions are

$$\Psi_1^{(1)} = \tfrac{1}{2}(1-N)(R - R^{-1})\sin^2\theta \tag{15a}$$

$$\Theta_1^{(1)} = \tfrac{1}{2}(1-N)\cos\theta[R^{-1} - \tfrac{3}{2}R^{-2} + \tfrac{1}{2}R^{-3}] \tag{15b}$$

$$C_1^{(1)} = Le\,\Theta_1^{(1)} \tag{15c}$$

and the second-order solutions are

$$\Psi_2^{(1)} = \tfrac{1}{4}(1-N)(1-N\,Le)\sin^2\theta\cos\theta\,[\tfrac{2}{3}R - \tfrac{3}{2} + R^{-1} - \tfrac{1}{6}R^{-2}] \tag{16a}$$

$$\begin{aligned}\Theta_2^{(1)} = {} & \frac{1}{4}(1-N)^2\left[-\frac{107}{180}R^{-1} + \frac{9}{8}R^{-2} - \frac{271}{504}R^{-3} - \frac{1}{6}R^{-3}\ln R + \frac{1}{140}R^{-5}\right] \\ & + \frac{1}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1} - \frac{15}{8}R^{-2} + \frac{109}{56}R^{-3} + \frac{1}{2}R^{-3}\ln R - \frac{3}{4}R^{-4} + \frac{5}{28}R^{-5}\right] \\ & + \frac{1}{4}(1-N)(1-N\,Le)\left[-\frac{1}{9}R^{-1} + \frac{3}{8}R^{-2} - \frac{17}{72}R^{-3} - \frac{1}{5}R^{-3}\ln R - \frac{1}{36}R^{-4}\right] \\ & + \frac{1}{4}(1-N)(1-N\,Le)\cos^2\theta\left[\frac{1}{3}R^{-1} - \frac{9}{8}R^{-2} + \frac{17}{24}R^{-3} + \frac{3}{5}R^{-3}\ln R + \frac{1}{12}R^{-4}\right]\end{aligned} \tag{16b}$$

$$C_2^{(1)} = Le^2(\Theta_{21}^{(1)} + \Theta_{22}^{(1)}) + Le(\Theta_{23}^{(1)} + \Theta_{24}^{(1)}) \tag{16c}$$

where the number in parentheses refers to the case under study, the first subscript represents the solution level for the specific case, and the second subscript the term in the expression of that solution. For example, $\Theta_{21}^{(1)}$ represents the first term in the expression of the second-order solution for case 1. This shorthand notation is very convenient in presenting our final results. As an indication of proper formulation and correct calculation, the above solution are reduced to those of Yamamoto [8] for pure thermal driven flow if one sets $N = 0$, i.e. in the absence of a concentration gradient.

*Case* 2. *A sphere of constant heat flux and mass flux*

The first-order solutions are

$$\Psi_1^{(2)} = \Psi_1^{(1)} \tag{17a}$$

$$\Theta_1^{(2)} = \tfrac{1}{2}(1-N)\cos\theta\,[R^{-1} - \tfrac{5}{4}R^{-2} + \tfrac{1}{2}R^{-3}] \tag{17b}$$

$$C_1^{(2)} = Le\,\Theta_1^{(2)} \tag{17c}$$

and the second-order solutions are

$$\Psi_2^{(2)} = \frac{1}{4}(1-N)(1-N\,Le)\sin^2\theta\cos\theta \times\left[\frac{2}{3}R-\frac{5}{4}+R^{-1}-\frac{5}{12}R^{-2}\right] \quad (18a)$$

$$\begin{aligned}\Theta_2^{(2)} = {} & \frac{1}{4}(1-N)^2\left[-\frac{5}{6}R^{-1}+\frac{15}{16}R^{-2} -\frac{209}{504}R^{-3}-\frac{1}{6}R^{-3}\ln R+\frac{1}{140}R^{-5}\right] \\ & +\frac{1}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1}-\frac{25}{16}R^{-2} +\frac{265}{168}R^{-3}+\frac{1}{2}R^{-3}\ln R-\frac{5}{8}R^{-4}+\frac{5}{28}R^{-5}\right] \\ & +\frac{1}{4}(1-N)(1-N\,Le)\left[-\frac{1}{9}R^{-1}+\frac{5}{16}R^{-2} -\frac{157}{1080}R^{-3}-\frac{1}{5}R^{-3}\ln R-\frac{5}{72}R^{-4}\right] \\ & +\frac{1}{4}(1-N)(1-N\,Le)\cos^2\theta\left[\frac{1}{3}R^{-1}-\frac{15}{16}R^{-2} +\frac{157}{360}R^{-3}+\frac{3}{5}R^{-3}\ln R+\frac{5}{24}R^{-4}\right] \quad (18b)\end{aligned}$$

$$C_2^{(2)} = Le^2(\Theta_{21}^{(2)}+\Theta_{22}^{(2)})+Le(\Theta_{23}^{(2)}+\Theta_{24}^{(2)}). \quad (18c)$$

*Case* 3. *A sphere of constant temperature and mass flux*

The first-order solutions are straightforward and can be readily obtained from the solutions of cases 1 and 2. However, the second-order solutions become somewhat involved due to the complication of mixed boundary conditions.

The first-order solutions are

$$\Psi_1^{(3)} = \Psi_1^{(1)} \quad (19a)$$

$$\Theta_1^{(3)} = \Theta_1^{(1)} \quad (19b)$$

$$C_1^{(3)} = C_1^{(2)} \quad (19c)$$

and the second-order solutions are

$$\begin{aligned}\Psi_2^{(3)} = {} & \frac{1}{4}(1-N)\sin^2\theta\cos\theta\left[\frac{2}{3}R-\frac{3}{2}+R^{-1}-\frac{1}{6}R^{-2}\right] \\ & -\frac{N\,Le}{4}(1-N)\sin^2\theta\cos\theta\left[\frac{2}{3}R-\frac{5}{4}+R^{-1}-\frac{5}{12}R^{-2}\right] \quad (20a)\end{aligned}$$

$$\begin{aligned}\Theta_2^{(3)} = {} & \frac{1}{4}(1-N)\left[-\frac{1}{9}R^{-1}+\frac{3}{8}R^{-2} -\frac{17}{72}R^{-3}-\frac{1}{5}R^{-3}\ln R-\frac{1}{36}R^{-4}\right] \\ & +\frac{1}{4}(1-N)\cos^2\theta\left[\frac{1}{3}R^{-1}-\frac{9}{8}R^{-2} +\frac{7}{24}R^{-3}+\frac{3}{5}R^{-3}\ln R+\frac{1}{12}R^{-4}\right] \\ & +\frac{N\,Le}{4}(1-N)\left[\frac{3}{8}R^{-1}-\frac{5}{16}R^{-2} -\frac{19}{144}R^{-3}+\frac{1}{5}R^{-3}\ln R+\frac{5}{72}R^{-4}\right] \\ & +\frac{N\,Le}{4}(1-N)\cos^2\theta\left[-\frac{1}{3}R^{-1}+\frac{15}{16}R^{-2} -\frac{19}{48}R^{-3}-\frac{3}{5}R^{-3}\ln R-\frac{5}{24}R^{-4}\right] \\ & +\frac{1}{4}(1-N)^2\left[-\frac{41}{72}R^{-1}+\frac{9}{8}R^{-2} -\frac{289}{504}R^{-3}-\frac{1}{6}R^{-3}\ln R+\frac{1}{56}R^{-5}\right] \\ & +\frac{1}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1}-\frac{15}{8}R^{-2}+\frac{115}{56}R^{-3} +\frac{1}{2}R^{-3}\ln R-\frac{3}{4}R^{-4}+\frac{1}{14}R^{-5}\right] \quad (20b)\end{aligned}$$

$$\begin{aligned}C_2^{(3)} = {} & \frac{Le}{4}(1-N)\left[-\frac{1}{9}R^{-1}+\frac{3}{8}R^{-2} -\frac{131}{540}R^{-3}-\frac{1}{5}R^{-3}\ln R-\frac{1}{36}R^{-4}\right] \\ & +\frac{Le}{4}(1-N)\cos^2\theta\left[\frac{1}{3}R^{-1}-\frac{9}{8}R^{-2} +\frac{131}{180}R^{-3}+\frac{3}{5}R^{-3}\ln R+\frac{1}{12}R^{-4}\right] \\ & +\frac{N\,Le^2}{4}(1-N)\left[\frac{1}{9}R^{-1}-\frac{5}{16}R^{-2} +\frac{157}{1080}R^{-3}+\frac{1}{5}R^{-3}\ln R+\frac{5}{72}R^{-4}\right] \\ & +\frac{N\,Le^2}{4}(1-N)\cos^2\theta\left[-\frac{1}{3}R^{-1}+\frac{15}{16}R^{-2} -\frac{157}{360}R^{-3}-\frac{3}{5}R^{-3}\ln R-\frac{5}{24}R^{-4}\right] \\ & +\frac{Le^2}{4}(1-N)^2\left[-\frac{5}{6}R^{-1}+\frac{15}{16}R^{-2} -\frac{209}{504}R^{-3}-\frac{1}{6}R^{-3}\ln R+\frac{1}{140}R^{-5}\right] \\ & +\frac{Le^2}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1}-\frac{25}{16}R^{-2}+\frac{265}{168}R^{-3} +\frac{1}{2}R^{-3}\ln R-\frac{5}{8}R^{-4}+\frac{5}{28}R^{-5}\right]. \quad (20c)\end{aligned}$$

*Case* 4. *A sphere of constant heat flux and concentration*

By the same manner described in case 3, the first-order solutions are

$$\Psi_1^{(4)} = \Psi_1^{(1)} \quad (21a)$$

$$\Theta_1^{(4)} = \Theta_1^{(2)} \quad (21b)$$

$$C_1^{(4)} = C_1^{(1)} \quad (21c)$$

and the second-order solutions are

$$\begin{aligned}\Psi_2^{(4)} = {} & \frac{1}{4}(1-N)\sin^2\theta\cos\theta\left[\frac{2}{3}R-\frac{5}{4}+R^{-1}-\frac{5}{12}R^{-2}\right] \\ & -\frac{N\,Le}{4}(1-N)\sin^2\theta\cos\theta\left[\frac{2}{3}R-\frac{3}{2}+R^{-1}-\frac{1}{6}R^{-2}\right] \quad (22a)\end{aligned}$$

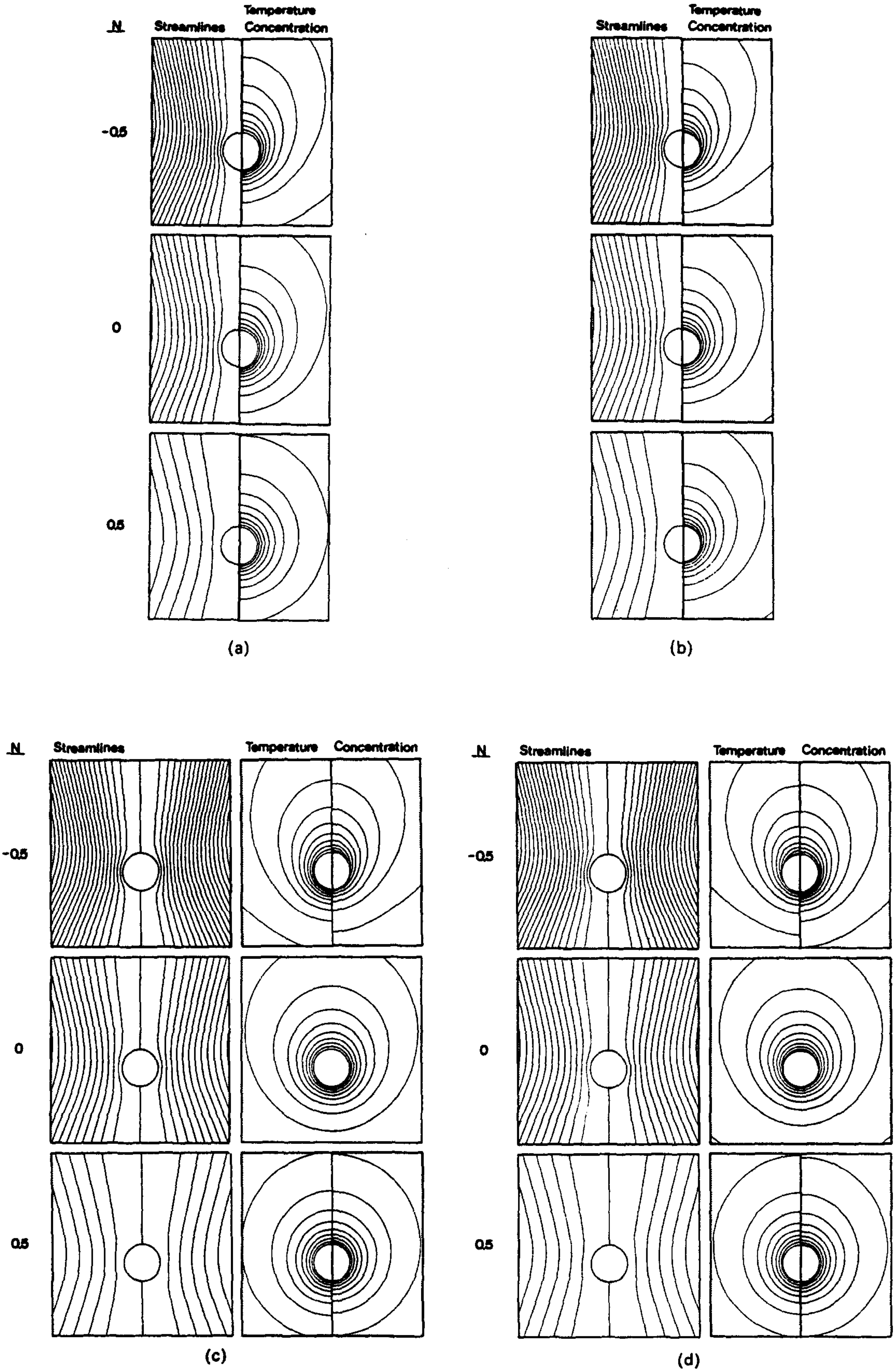


FIG. 1. Steady-state flow, temperature and concentration fields for $Ra = 1$ and $Le = 1$ ($\Delta\Psi = 0.2$, $\Delta\Theta = \Delta C = 0.1$): (a) case 1; (b) case 2; (c) case 3; (d) case 4.

$$\begin{aligned}
\Theta_2^{(4)} &= \frac{1}{4}(1-N)\left[-\frac{1}{9}R^{-1}+\frac{5}{16}R^{-2}\right.\\
&\quad\left.-\frac{157}{1080}R^{-3}-\frac{1}{5}R^{-3}\ln R-\frac{5}{72}R^{-4}\right]\\
&\quad+\frac{1}{4}(1-N)\cos^2\theta\left[\frac{1}{3}R^{-1}-\frac{15}{16}R^{-2}\right.\\
&\quad\left.+\frac{157}{360}R^{-3}+\frac{3}{5}R^{-3}\ln R+\frac{5}{24}R^{-4}\right]\\
&\quad+\frac{N\,Le}{4}(1-N)\left[\frac{1}{9}R^{-1}-\frac{3}{8}R^{-2}\right.\\
&\quad\left.+\frac{131}{540}R^{-3}+\frac{1}{5}R^{-3}\ln R+\frac{1}{36}R^{-4}\right]\\
&\quad+\frac{N\,Le}{4}(1-N)\cos^2\theta\left[-\frac{1}{3}R^{-1}+\frac{9}{8}R^{-2}\right.\\
&\quad\left.-\frac{131}{180}R^{-3}-\frac{3}{5}R^{-3}\ln R-\frac{1}{12}R^{-4}\right]\\
&\quad+\frac{1}{4}(1-N)^2\left[-\frac{5}{6}R^{-1}+\frac{15}{16}R^{-2}\right.\\
&\quad\left.-\frac{209}{504}R^{-3}-\frac{1}{6}R^{-3}\ln R+\frac{1}{140}R^{-5}\right]\\
&\quad+\frac{1}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1}-\frac{25}{16}R^{-2}+\frac{265}{168}R^{-3}\right.\\
&\quad\left.+\frac{1}{2}R^{-3}\ln R-\frac{5}{8}R^{-4}+\frac{5}{28}R^{-5}\right]
\end{aligned} \tag{22b}$$

$$\begin{aligned}
C_2^{(4)} &= \frac{Le}{4}(1-N)\left[-\frac{3}{8}R^{-1}+\frac{5}{16}R^{-2}\right.\\
&\quad\left.+\frac{19}{144}R^{-3}-\frac{1}{5}R^{-3}\ln R-\frac{5}{72}R^{-4}\right]\\
&\quad+\frac{Le}{4}(1-N)\cos^2\theta\left[\frac{1}{3}R^{-1}-\frac{15}{16}R^{-2}\right.\\
&\quad\left.+\frac{19}{48}R^{-3}+\frac{3}{5}R^{-3}\ln R+\frac{5}{24}R^{-4}\right]
\end{aligned}$$

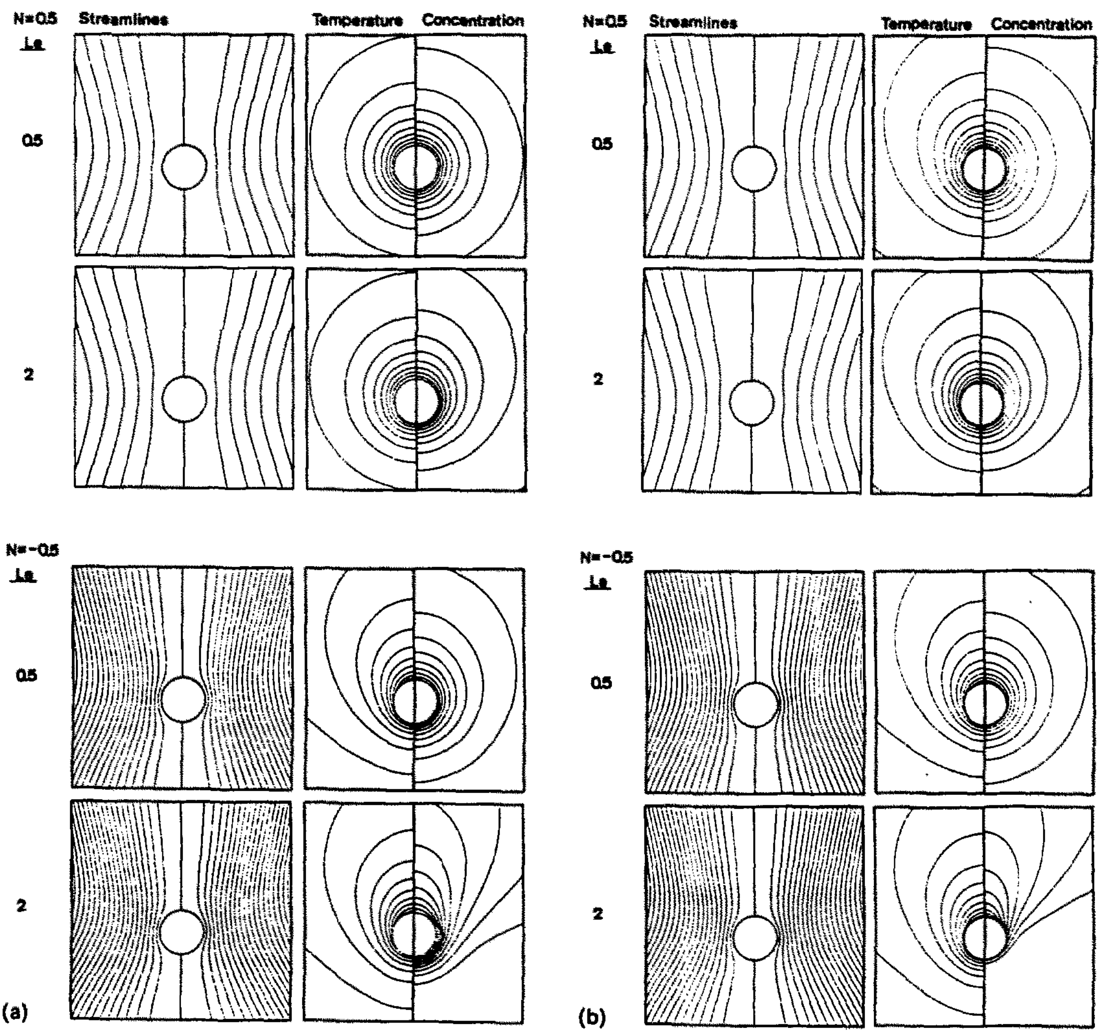


FIG. 2. Steady-state flow, temperature and concentration fields for $Ra = 1$ and $N = 0.5$ and $-0.5$ ($\Delta\Psi = 0.2$, $\Delta\Theta = \Delta C = 0.1$): (a) case 1; (b) case 2; (c) case 3; (d) case 4.

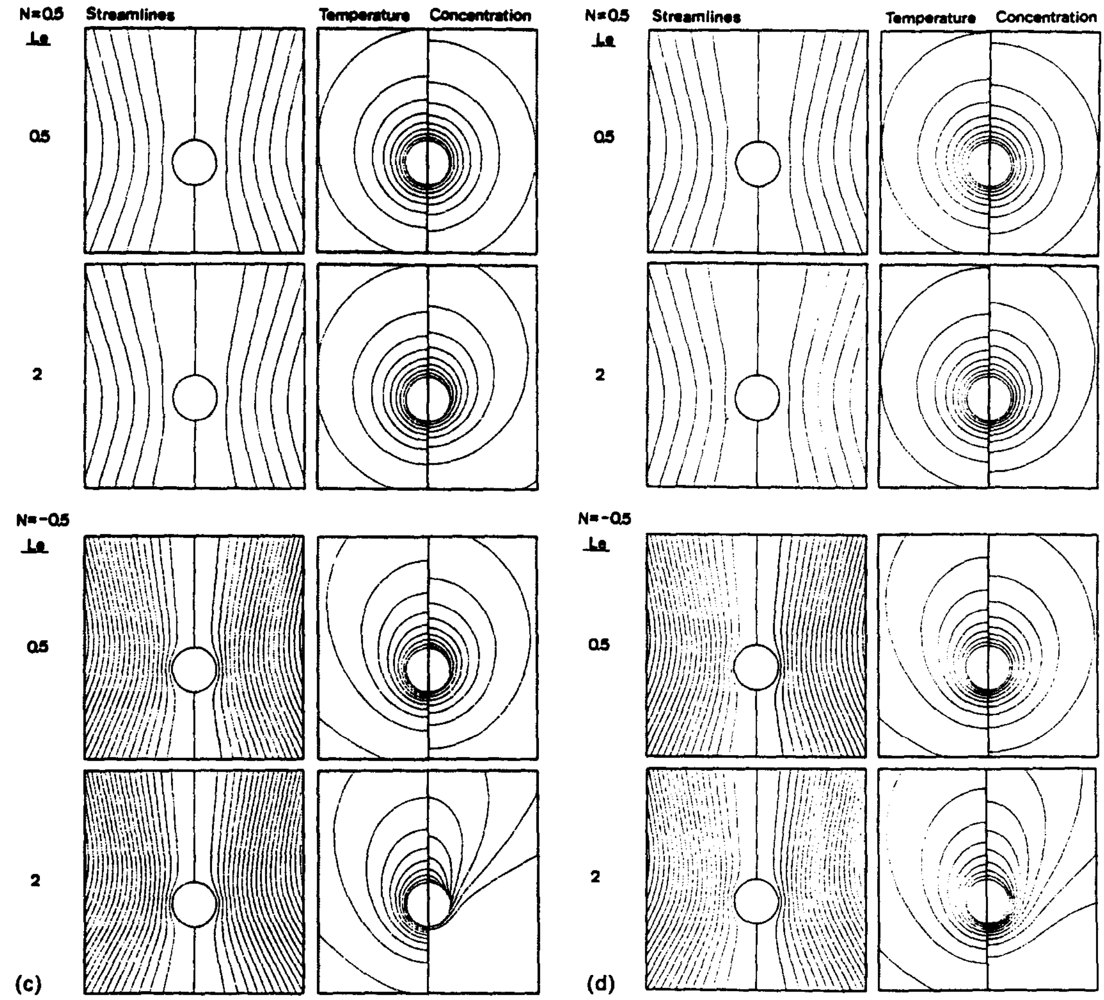


FIG. 2. *(continued)*

$$
\begin{aligned}
&+\frac{N\,Le^2}{4}(1-N)\left[\frac{1}{9}R^{-1}-\frac{3}{8}R^{-2}\right.\\
&\left.+\frac{17}{72}R^{-3}+\frac{1}{5}R^{-3}\ln R+\frac{1}{36}R^{-4}\right]\\
&+\frac{N\,Le^2}{4}(1-N)\cos^2\theta\left[-\frac{1}{3}R^{-1}+\frac{9}{8}R^{-2}\right.\\
&\left.-\frac{17}{24}R^{-3}-\frac{3}{5}R^{-3}\ln R-\frac{1}{12}R^{-4}\right]\\
&+\frac{Le^2}{4}(1-N)^2\left[-\frac{41}{72}R^{-1}+\frac{9}{8}R^{-2}\right.\\
&\left.-\frac{289}{504}R^{-3}-\frac{1}{6}R^{-3}\ln R+\frac{1}{56}R^{-5}\right]\\
&+\frac{Le^2}{4}(1-N)^2\cos^2\theta\left[\frac{1}{2}R^{-1}-\frac{15}{8}R^{-2}+\frac{115}{56}R^{-3}\right.\\
&\left.+\frac{1}{2}R^{-3}\ln R-\frac{3}{4}R^{-4}+\frac{1}{14}R^{-5}\right]. \qquad (22c)
\end{aligned}
$$

For cases 1 and 2, it is noticed that equations (2) and (3) as well as boundary conditions (5)–(7) take an identical form if $Le = 1$. Therefore, solutions for the temperature field are exactly the same as those for the concentration field. However, this is not the case for cases 3 and 4 due to differences in the boundary conditions.

To examine the roles played by the two new parameters, i.e. the buoyancy ratio $N$ and Lewis number $Le$, results are plotted in Figs. 1 and 2 for a fixed $Le$ and a fixed $N$, respectively. As stated earlier, the sign of the buoyancy ratio determines if the concentration gradient is against or in favor of the thermal buoyancy driven flow. This is clearly demonstrated in Fig. 1 where the flow and temperature fields of $N = 0$ are also included for comparison. For $N < 0$, the concentration gradients assist the flow, while they suppress it for $N > 0$. As a result of this interaction, it is noticed that the warm, high concentration region shifts upwards for $N < 0$ and downwards for $N > 0$.

For a fixed $N$, an increase in the Lewis number has a more pronounced effect on the concentration field than it does on the flow and temperature fields (Fig. 2). It is also noted that an increase in the Lewis number has further assisted the thermal buoyancy driven flow for $N < 0$, and suppressed it for $N > 0$. In addition, the warm, high concentration region also shifts upwards for $N < 0$ and downwards for $N > 0$.

It should be pointed out, however, that the solutions thus obtained are expected to be valid in the diffusion dominated regime. For a situation when the Lewis number is large, the solutions may not hold due to the resulting strong convection (even at a small $Ra$). This increase in the strength of the convective flow can also be observed in Fig. 2.

To summarize, the flow and temperature fields are significantly modified by the inclusion of mass transfer effects. In the presence of a concentration gradient, flow can be either aided or retarded, depending on the sign of the buoyancy ratio $N$. The Lewis number is observed to have a stronger influence on the concentration field than it does on the flow and temperature fields. In addition, it amplifies the results produced by the buoyancy ratio.

*Acknowledgement*—The support of this work by the U.S. Nuclear Regulatory Commission and the Computer Center at the Colorado State University is greatly appreciated.

## REFERENCES


1. D. A. Nield, Onset of thermohaline convection in a porous medium, *Water Resour. Res.* **4**, 553–560 (1968).
2. A. Bejan and K. R. Khair, Heat and mass transfer by natural convection in a porous medium, *Int. J. Heat Mass Transfer* **28**, 909–918 (1985).
3. O. V. Trevisan and A. Bejan, Natural convection with combined heat and mass transfer buoyancy effects in a porous medium, *Int. J. Heat Mass Transfer* **28**, 1597–1611 (1985).
4. O. V. Trevisan and A. Bejan, Mass and heat transfer by natural convection in a vertical slot filled with porous medium, *Int. J. Heat Mass Transfer* **29**, 403–415 (1986).
5. O. V. Trevisan and A. Bejan, Mass and heat transfer by high Rayleigh number convection in a porous medium heated from below, *Int. J. Heat Mass Transfer* **30**, 2341–2356 (1987).
6. D. Poulikakos, On buoyancy induced heat and mass transfer from a concentrated source in an infinite porous medium, *Int. J. Heat Mass Transfer* **28**, 621–629 (1985).
7. A. Bejan, Natural convection in an infinite porous medium with a concentrated heat source, *J. Fluid Mech.* **89**, 97–107 (1978).
8. K. Yamamoto, Natural convection about a heated sphere in a porous medium, *J. Phys. Soc. Japan* **37**, 1164–1166 (1974).


*Int. J. Heat Mass Transfer.* Vol. 33, No. 1, pp. 215–218, 1990
Printed in Great Britain
0017-9310/90 $3.00+0.00


# Radiative configuration factors from cylinders to coaxial axisymmetric bodies

C. Saltiel† and M. H. N. Naraghi

Department of Mechanical Engineering, Manhattan College, Riverdale, NY 10471, U.S.A.

(*Received* 10 *April* 1989 *and in final form* 13 *June* 1989)

## INTRODUCTION

Many practical engineering applications in radiative heat transfer require the evaluation of geometric configuration factors between a cylinder and a coaxial axisymmetric body, such as pipe exhaust systems (including a rocket and its plume) and annular radiative fins. Although view factors for such geometries resist closed-form solutions, the number of required integrations for a numerical calculation could be reduced substantially if analytic expressions are obtained for the configuration factors between differential elements of the axisymmetric body and the cylinder. In this note, exact solutions are derived for shape factors between differential elements of arbitrary orientation and cylinders. Using these derivations, we illustrate the calculation of view factors between cylinders and general coaxial bodies via a method in which only a single numerical integration need be performed [1].

### *View factor from a differential element to a cylinder*

The configuration shown in Fig. 1 depicts a differential area $dA_2$ and a cylinder $Cy$. The unit normal vector to $dA_2$ lies in the $y$–$z$ plane. If the angle $\theta$ is less than $\tan^{-1}[HP/(P^2-1)]$, where $H = h/r$ and $P = p/r$ are the dimensionless cylinder height and distance from the differential element to the axis of symmetry, respectively, the contour of the section of the cylinder which is visible to the differential area consists of four curves: two vertical lines $\overline{GB}$ and $\overline{DE}$, the circular arc $\overline{EFG}$, and the elliptic arc $\overline{BCD}$. The view factor from the differential area to the cylinder can be determined by integrating over this contour [2]. Since the line integral over arc $\overline{BCD}$ is identical to that over the horizontal $\overline{BD}$, only the contour $\overline{BDEFGB}$ need be evaluated. The curves describing this contour can be expressed in the non-dimensional form as

| | |
|---|---|
| $\overline{GB}$ | $X = \sqrt{(P^2-1)}/P, \quad Y = 1/P, \quad Z = Z$ |
| $\overline{DE}$ | $X = -\sqrt{(P^2-1)}/P, \quad Y = 1/P, \quad Z = Z$ |
| $\overline{DB}$ | $X = X, \quad Y = 1/P, \quad Z = (P^2-1)/P\tan\theta$ |
| $\overline{EFG}$ | $X = \sin\beta, \quad Y = \cos\beta, \quad Z = H.$ |

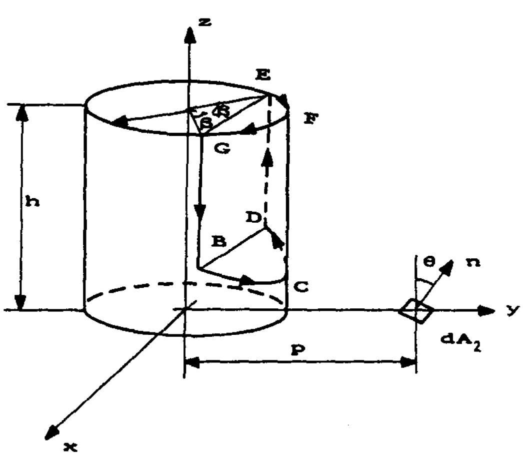


Fig. 1. Cylinder and differential element configuration.

† Present address: Department of Mechanical Engineering, University of Florida, Gainesville, FL 32611, U.S.A.